stresses on the end surfaces, which are not taken into account in the calculation so that the rather close agreement observed in this particular instance may be partly fortuitous.

The application of the similarity method determines the distortion coefficients of the type a) and type b) assemblies quite independently of one another. Since the two types are found to have appreciably different coefficients, a direct comparison, e. g. by balancing a steel assembly of type a) against one of type b), is now able to provide an additional check of the overall accuracy of the procedure. The results of experiments on these lines are shown in Fig. 4, which compares the values of the distortion factors of a type b) assembly derived in two independent ways:

i) by direct application of the similarity method to the type b) assembly, and

ii) by comparison of the same type b) assembly with assemblies of type a), the distortion factors of which had previously been determined by direct application of the similarity method.

It will be seen that the results obtained by the two methods are practically indistinguishable; the actual mean values of several determinations of the distor-

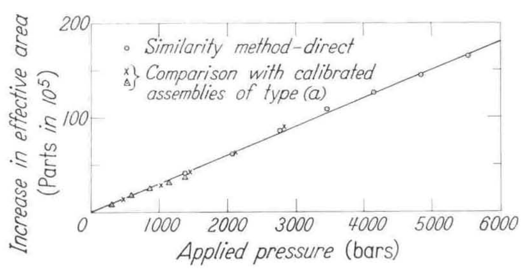

Fig. 4. Distortion factor of assembly of type b determined by two methods

tion factor for the type b) assembly are $3.0_2 \times 10^{-7}$ and $2.9_5 \times 10^{-7}$/bar for procedures i) and ii) respectively. This independent check thus supports the estimates of accuracy put forward in the foregoing section.

d) Practical applications

Once the effective area of a pressure balance assembly has been measured in absolute terms as a function of pressure over a given range, it is possible to calibrate almost any other assembly covering the same range, and using the same pressure transmitting fluid, by the process of direct balancing. In the course of the present investigation a large number of individual balances of different patterns have been calibrated, including many for other users. Balances involving piston-cylinder assemblies of types a) and b) — Fig. 2 — have already been discussed. These show, for a given fluid, fairly consistent distortion coefficients, typified by the values given above in sections 4 b) and 4 c). In such cases, it may be sufficient for many purposes to take an average figure as typical of assemblies of a given pattern.

Another type of balance in common use, of which a considerable number have been calibrated, is that employing a simple piston-cylinder assembly consisting of a bronze cylinder combined with a steel piston. This type also exhibits fair consistency as regards dependence of effective area upon pressure, the distortion coefficient being about 8×10^{-7}/bar.

Calibrations have also been made of a number of differential piston-cylinder assemblies of the well known form shown diagrammatically in Fig. 5. In this type of assembly the actual effective area is the difference between the effective areas of the two constituent piston-cylinder combinations, the upper combination being varied in diameter to suit the desired pressure range. The considerations leading to the approximate equation (2.6) may easily be extended to include this differential type of assembly (e. g. ZHOKOVSKII 1960) and lead to the expectation of a distortion coefficient in the region 3 to 4×10^{-7}/bar, with a gradual decrease as the diameter of the upper unit is reduced. Experience at the National Physical Laboratory so far has indicated, however, that this

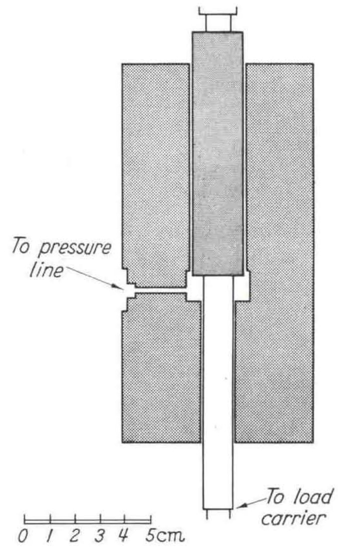

Fig. 5. Diagram of differential piston-cylinder assembly

type of assembly does not exhibit the kind of consistency found in the case of the simple piston-cylinder assemblies. In a group of ten such differential assemblies coefficients ranging from about zero to 11×10^{-7}/bar were found, with no indication of any regular dependence on the constituent piston diameters. This may be due to the fact that in many cases the effective area is the difference between two much larger areas so that the effect of any abnormality on the part of either of the constituent piston-cylinder combinations may be considerably magnified. It could also be associated in part with the difficulty of constructing such assemblies with the two cylinders exactly coaxial. Whatever the explanation, however, it seems that each assembly of this type requires individual calibration and that the assignment of typical values of the distortion coefficient, or reliance on calculated values, would not be satisfactory in this case.

5. The Flow Method

a) Principle of the method

The flow method was developed in order to provide an independent check of the changes of effective area of a pressure balance assembly determined by the similarity method, by means which would be independent of the considerations on which the similarity method is based, but which would still depend entirely on the properties of the assembly itself without reference to other standards of pressure.

The principle used is to introduce a deliberate and accurately measurable initial change of effective area — by varying the diameter of one of the components of the assembly — which is made to serve as a reference quantity in terms of which the additional changes of effective area due to pressure may be calculated from measurements of other quantities which vary with the applied pressure.

The procedure used is actually only one of a class of possible methods, of which others will be mentioned below. In the form adopted the rates of flow of the pressure-transmitting fluid through the interspace between the piston and cylinder are measured, at a series of applied pressures, using two alternative pistons having an accurately known difference of daimeter. A simple relation may then be developed connecting the changes of effective area due to distortion with the initial change due to the different piston diameter, and the rates of flow corresponding to the two pistons.

Two other methods of the same general nature, but not depending on flow measurement, were considered and some preliminary experiments carried out. In the first case the quantity measured was the rate of retardation of the rotation speed of the piston and loading weights due to fluid friction in the clearance between piston and cylinder, corresponding to the two piston diameters. It was found, however, that the contribution due to air friction on the rotating load system was an important factor, and rather elaborate measures would have been necessary to eliminate this effect. In the second case the intention was to compare the electrical capacitances of the piston-cylinder assembly corresponding to the two piston diameters. This method, on which so far only very preliminary trials have been made, would very likely repay further exploration, but a knowledge of the pressure dependence of the dielectric constant of the transmitting fluid would be required to complete the reduction of the experimental data.

b) Theory of the flow method

The main problem in the theory of the method is to establish a reasonably simple connection between the measured rates of flow of the pressure transmitting fluid and the corresponding changes of effective area at the same applied pressures.

To introduce the variation of effective area with pressure we adopt the formal expression (2.5) of section 2 b, in which the only term dependent upon h is the integral $\frac{2}{rP}\int_0^P h\,dp$. The remaining variable term, $P\,(3\,\sigma - 1)/E$, is a small part of the total, and it has already been seen that the assumption on which the derivation of this term is based is unlikely to lead to appreciable error.

Denoting by Q the volume velocity of the fluid through any section of the annular gap, and $\eta\,(x)$ the coefficient of viscosity of the fluid at the axial distance x, it is easily shown that, under conditions of viscous flow,

$$\frac{3\,Q}{4\,\pi r} = -\frac{dp}{dx}\,\frac{h^3}{\eta}\ * \qquad (5.1)$$

* To avoid unnecessarily complicating the notation we ignore variations of the density of the fluid with pressure, as these are very unimportant compared with the variations in the coefficient of viscosity.

and by direct integration, we have

$$\frac{3\,Q}{4\,\pi r} = \int_0^P \frac{h^3}{\eta}\,dp\ . \qquad (5.2)$$

In order to exhibit the direct relation between Q and $\int_0^P h\,dp$ in a suitable form we may integrate equation (5.1) by a different route, whence we obtain

$$\left(\frac{3\,Q}{4\,\pi r}\right)^{\frac{1}{3}} = -\int_0^P h\,dp \Bigg/ \int_0^P \left(\eta\,\frac{dx}{dp}\right)^{\frac{1}{3}} dp\ . \qquad (5.3)$$

This equation shows that the factor relating $Q^{\frac{1}{3}}$ to $\int_0^P h\,dp$ is a function only of the pressure distribution in the interspace between piston and cylinder, and is not explicitly dependent on h. This suggests the possibility that $\int_0^P \left(\eta\,\frac{dx}{dp}\right)^{\frac{1}{3}} dp$ may not vary very much for a moderate change in the initial diameter of the piston.

Re-arranging equations (5.2) and (5.3), and writing for brevity

$$\chi = \left(\frac{3\,Q}{4\,\pi r}\right)^{\frac{1}{3}} \quad \text{and} \quad I = -\int_0^P \left(\eta\,\frac{dx}{dp}\right)^{\frac{1}{3}} dp\ ,$$

we have

$$\int_0^P h\,dp = \chi\,I\ ;\ I = l^{\frac{1}{3}} \int_0^P h\,dp \Bigg/ \left(\int_0^P \frac{h^3}{\eta}\,dp\right)^{\frac{1}{3}}\ . \qquad (5.4)$$

The second of these equations provides the basis for the calculation of the integral factor I, connecting the required changes of effective area with the experimentally determined rates of flow.

Before considering further the evaluation of the integral I, it is convenient to convert the formal equations connecting the changes of effective area with the quantities χ and I to a form suitable for application to the experimental data. Proceeding from equation (2.5) and using suffixes 1, 2 where necessary to distinguish the two piston diameters, and denoting by δr the value of $(r_1 - r_2)$ we have

$$A_{P,1} = \pi r_1^2 \left[1 + \frac{P}{E}\,(3\,\sigma - 1) + \frac{2}{r_1\,P}\int_0^P h_1\,dp_1\right],$$

$$A_{P,2} = \pi r_2^2 \left[1 + \frac{P}{E}\,(3\,\sigma - 1) + \frac{2}{r_2\,P}\int_0^P h_2\,dp_2\right]$$

whence, ignoring terms of the second order of small quantities,

$$A_{P,1} + A_{P,2} = 2\,\pi r_1^2 \Bigg[1 + \frac{P}{E}\,(3\,\sigma - 1) - \frac{\delta r}{r} + \\ + \frac{1}{r\,P}\left(\int_0^P h_1\,dp_1 + \int_0^P h_2\,dp_2\right)\Bigg],$$

and

$$A_{P,1} - A_{P,2} = 2\,\pi r_1^2 \left[\frac{\delta r}{r} + \frac{1}{r\,P}\left(\int_0^P h_1\,dp_1 - \int_0^P h_2\,dp_2\right)\right].$$